Coherent and incoherent pair creation by a photon in oriented single crystal

V. N. Baier and V. M. Katkov
Budker Institute of Nuclear Physics,
Novosibirsk, 630090, Russia

February 2, 2008

Abstract

The new approach is developed for study of electron-positron pair production by a photon in oriented single crystal. It permits indivisible consideration of both coherent and incoherent mechanisms of pair creation and includes the action of field of axis (or plane) as well as the multiple scattering of particles of the created pair (the Landau-Pomeranchuk-Migdal (LPM) effect). From obtained integral probability of pair creation, it follows that multiple scattering appears only for relatively low energy of photon, while at higher photon energy the field action excludes the LPM effect. The found probabilities agree quite satisfactory with recent CERN experiment.

It is known that mechanism of pair production by a photon in oriented single crystal differs substantially from the mechanism of independent pair creation at separate centers acting in an amorphous medium (the Bethe-Heitler mechanism). In crystal the coherent interaction of a photon with many centers occurs. Under some generic assumptions the general theory of the coherent pair creation mechanism was developed in [1]. If the photon angle of incidence ϑ_0 (the angle between photon momentum $\mathbf{k}$ and the axis (or plane)) is small $\vartheta_0 \ll V_0/m$, where V_0 is the characteristic scale of the potential, the field E of the axis (or plane) can be considered constant over the pair formation length and the constant-field approximation is valid. In this case the behavior of pair production probability is determined by the parameter

$$\kappa = \frac{\omega}{m}\frac{E}{E_0}, \tag{1}$$

where ω is the photon energy, m is the electron mass, $E_0 = m^2/e = 1.32 \cdot 10^{16}$ V/cm is the critical field, the system $\hbar = c = 1$ is used. The very important feature of coherent pair creation mechanism is the strong enhancement of its probability at high energies (from factor ~ 10 for main axes in crystals of heavy elements like tungsten to factor ~ 160 for diamond) comparing with the Bethe-Heitler mechanism. If $\vartheta_0 \gg V_0/m$ the theory passes over to the coherent pair production theory (see e.g. [3], Sec.13.1). Side by side with coherent mechanism the incoherent mechanism of pair creation is acting. In oriented crystal this mechanism changes also with respect to an amorphous medium [2]. The details of theory and description of experimental study of pair production which confirms the mentioned enhancement can be found in [3]. The item continues to attract attention [4] and new experiments are performed recently [5], [6]. In high-energy region the multiple scattering of particles of created pair (the Landau-Pomeranchuk-Migdal (LPM) effect) is influenced in probability of pair creation. This paper is devoted to study of the problem using the new theoretical approach..

The properties of pair creation process are connected directly with details of motion of particles of created pair. The momentum transfer from a particle to a crystal we present in a form $\mathbf{q} =< \mathbf{q} > +\mathbf{q}_s$, where $< \mathbf{q} >$ is the mean value of momentum transfer calculated with averaging over thermal(zero) vibrations of atoms in a crystal. The motion of particles in an averaged potential of crystal, which corresponds to the momentum transfer $< \mathbf{q} >$, determines the coherent mechanism of pair creation. The term $\mathbf{q}_s$ is attributed to the random collisions of particle which define the incoherent mechanism of discussed process. Such random collisions we will call "scattering" since $< \mathbf{q}_s >= 0$. If the formation length of pair creation process is large with respect to distances between atoms forming the axis, the additional averaging over the atom position should be performed.

Here we consider case $\vartheta_0 \ll V_0/m$. Than the distance from axis $\boldsymbol{\varrho}$ as well as the transverse field of the axis can be considered as constant over the formation length. The process of the electron-positron pair creation in this case is one of interesting applications of the theory of propagation of a photon in a medium in presence of

an external field [7]. In the problem under consideration we have both the dense matter with strong multiple scattering and high field of crystal axis.

For an axial orientation of crystal the ratio of the atom density $n(\varrho)$ in the vicinity of an axis to the mean atom density n_a is

$$\frac{n(x)}{n_a} = \xi(x) = \frac{x_0}{\eta_1} e^{-x/\eta_1}, \tag{2}$$

where

$$x_0 = \frac{1}{dn_a a_s^2}, \quad \eta_1 = \frac{2u_1^2}{a_s^2}, \quad x = \frac{\varrho^2}{a_s^2}, \tag{3}$$

Here ϱ is the distance from axis, u_1 is the amplitude of thermal vibration, d is the mean distance between atoms forming the axis, a_s is the effective screening radius of the axis potential (see Eq.(9.13) in [3])

$$U(x) = V_0 \left[\ln\left(1 + \frac{1}{x+\eta}\right) - \ln\left(1 + \frac{1}{x_0+\eta}\right)\right]. \tag{4}$$

The local value of parameter $\kappa(x)$ (see Eq.(1)) which determines the probability of pair creation in the field Eq.(4 is

$$\kappa(x) = -\frac{dU(\varrho)}{d\varrho}\frac{\omega}{m^3} = \kappa_s \frac{2\sqrt{x}}{(x+\eta)(x+\eta+1)}, \quad \kappa_s = \frac{V_0\omega}{m^3 a_s} \equiv \frac{\omega}{\omega_s}. \tag{5}$$

The parameters of the axial potential for the ordinarily used crystals are given in Table 9.1 in [3]. The particular calculation below will be done for tungsten crystals studied in [5]. The relevant parameters are given in Table 1.

In an amorphous medium (or in crystal in the case of random orientation) the LPM effect becomes essential for heavy elements at the characteristic photon energy $\omega_e \sim 10$ TeV [8] and this value is inversely proportional to the density. In the aligned case the ratio $\xi(0)$ Eq.(2) may attain the magnitude $\xi(0) \sim 10^3$ in cold crystals. So the characteristic photon energy becomes $\omega_0 = \omega_e/\xi(0) \sim 10$ GeV. It is useful to compare the characteristic energy ω_0 with known threshold energy ω_t for which the probability of pair creation in the axis field becomes equal to the Bethe-Maximon probability, see Sec.12.2 and Table 12.1 in [3]. For small value of the parameter κ the probability of coherent pair creation is (see Eq.(12.11) in [3])

$$W^F = \frac{9}{32}\sqrt{\frac{\pi}{2}}\frac{\alpha m^2}{\omega x_0}\frac{\kappa_m^2}{\sqrt{-\kappa_m''}}\exp(-8/3\kappa_m). \tag{6}$$

Here κ_m is the maximal value of the parameter $\kappa(x)$ Eq.(5) (which defines the value of ω_t)

$$\kappa_m = \kappa(x_m), \quad x_m = \frac{1}{6}\left(\sqrt{1+16\eta(1+\eta)} - 1 - 2\eta\right), \quad \kappa_m'' = \kappa''(x_m) \tag{7}$$

We present it in the form $\kappa_m = \omega/\omega_m$. Than we find that $\omega_t \sim \omega_m \sim \omega_0$ for main axes of crystals of heavy elements. So at $\omega \sim \omega_t$ all the discussed effects are simultaneously essential in these crystals. In crystals of elements with intermediate Z (Ge, Si, diamond) the ratio $\omega_t/\omega_m \sim 1$ but $\omega_m/\omega_0 \ll 1$. So, one can neglect the LPM effect at $\omega \sim \omega_t$.

At $\omega \ll \omega_t$ the pair creation integral cross section for incoherent mechanism in oriented crystal has the form (see Eq.(26.30) in [3])

$$
\begin{aligned}
&\sigma_p = \frac{28Z^2\alpha^3}{9m^2}\left[L_0 - \frac{1}{42} - h\left(\frac{u_1^2}{a^2}\right)\right], \quad L_0 = \ln(ma) + \frac{1}{2} - f(Z\alpha), \\
&h(z) = -\frac{1}{2}\left[1 + (1+z)e^z\mathrm{Ei}(-z)\right], \quad a = \frac{111Z^{-1/3}}{m}, \\
&f(\xi) = \mathrm{Re}\left[\psi(i + i\xi) - \psi(1)\right] = \sum_{n=1}^{\infty}\frac{\xi^2}{n(n^2+\xi^2)},
\end{aligned}
\tag{8}
$$

where $\psi(z)$ is the logarithmic derivative of the gamma function, $\mathrm{Ei}(z)$ is the integral exponential function, $f(\xi)$ is the Coulomb correction. The cross section differs from the Bethe-Maximon cross section only by the term $h(u_1^2/a^2)$ which reflects the nongomogeneity of atom distribution in crystal. For $u_1 \ll a$ one has $h(u_1^2/a^2) \simeq -(1+C)/2 + \ln(a/u_1)$, $C = 0.577..$ and so this term characterizes the new value of upper boundary of impact parameters u_1 contributing to the value $< \mathbf{q}_s^2 >$ instead of screening radius a in an amorphous medium.

The influence of axis field on the incoherent pair creation process begins when ω becomes close to ω_m. For small values of the parameter κ_m the correction to the cross section Eq.(7) is (see Eq.(7.137) in [3])

$$
\begin{aligned}
&\Delta\sigma_p = \frac{176}{175}\frac{Z^2\alpha^3}{m^2}\overline{\kappa^2}\left(L_u - \frac{1789}{1980}\right), \\
&\overline{\kappa^2} = \int_0^{\infty}\frac{dx}{\eta_1}e^{-x/\eta_1}\kappa^2(x), \quad L_u = L_0 - h\left(\frac{u_1^2}{a^2}\right).
\end{aligned}
\tag{9}
$$

The coherent and incoherent contribution to pair creation can separated also for $\kappa_m \gg 1$ ($\omega \gg \omega_m$). In this case one can use the perturbation theory in calculation of the probability of incoherent process and neglect the LPM effect because of domination of the coherent contribution and additional suppression (by the axis field) the incoherent process. In this case the local cross section of pair creation has the form (see Eq.(7.138) in [3])

$$
\sigma_p(x) = \frac{8Z^2\alpha^3\Gamma^3(1/3)}{25m^2(3\kappa(x))^{2/3}\Gamma(2/3)}\left(L_u + 0.4416 + \frac{1}{3}\ln\kappa(x)\right). \tag{10}
$$

Averaging the function $(\kappa(x))^{-2/3}$ and $\ln\kappa(x)(\kappa(x))^{-2/3}$ over x according with Eq.(9) one can find the effective value of upper boundary of the transverse momentum transfer ($\propto m\kappa_m^{1/3}$ instead of m) which contributes to the value $< \mathbf{q}_s^2 >$.

Using the obtained results we determine the effective logarithm L by means of interpolation procedure

$$L = L_0 g, \quad g = 1 + \frac{1}{L_0}\left[-\frac{1}{42} - h\left(\frac{u_1^2}{a^2}\right) + \frac{1}{3}\ln\left(\frac{6 - 3\kappa_m^2 + 3\kappa_m^3}{6 + \kappa_m^2}\right)\right]. \tag{11}$$

Let us introduce the local characteristic energy

$$\omega_c(x) = \frac{m}{4\pi Z^2\alpha^2\lambda_c^3 n(x) L} = \frac{\omega_e(n_a)}{\xi(x) g} = \frac{\omega_0}{g} e^{x/\eta_1}, \tag{12}$$

where $\lambda_c = 1/m$. In this notations the local probability for small values of κ_m and ω/ω_0 has a form (see Eq.(7.137) in [3] and Eq.(2.23) in [8])

$$W(x) = \frac{7}{9\pi}\frac{\alpha m^2}{\omega_c(x)}\left[1 + \frac{396}{1225}\kappa^2(x) - \frac{3312}{2401}\frac{\omega^2}{\omega_c^2(x)}\right], \tag{13}$$

where the term with $\kappa^2(x)$ arises due to the field action and the term with $\omega^2/\omega_c^2(x)$ reflects influence of multiple scattering (the LPM effect). Averaging this expression over x we have

$$\begin{gathered}
\int_0^\infty \frac{dx}{x_0}\frac{1}{\omega_c(x)} = \frac{g}{\omega_0}\frac{\eta_1}{x_0} = \frac{g}{\omega_e(n_a)}, \quad \int_0^\infty \frac{dx}{x_0}\frac{1}{\omega_c^3(x)} = \frac{g}{\omega_e(n_a)}\frac{g^2}{3\omega_0^2}, \\
\int_0^\infty \frac{dx}{x_0}\frac{\kappa^2(x)}{\omega_c(x)} = \frac{g}{\omega_e(n_a)}\overline{\kappa^2}, \\
W \equiv \overline{W(x)} = W_0 g\left[1 + \frac{396}{1225}\overline{\kappa^2} - \frac{1104}{2401}\left(\frac{\omega g}{\omega_0}\right)^2\right],
\end{gathered} \tag{14}$$

where W_0 is

$$W_0 = \frac{7}{9}\frac{\alpha m^2}{\pi\omega_e(n_a)} = \frac{28}{9}\frac{Z^2\alpha^3}{m^2} n_a L_0. \tag{15}$$

The general expression for integral probability of pair creation by a photon under the simultaneous action of multiple scattering and an external constant field was obtained in [7] (see Eqs.(2.14) and (1.12)). For further analysis and numerical calculation it is convenient to turn the contour of integration over t at the angle $-i\pi/4$. We obtain after substitution $t \to \sqrt{2}t$

$$\begin{gathered}
W = \frac{\alpha m^2}{2\pi\omega}\int_0^1 \frac{dy}{y(1-y)}\int_0^{x_0}\frac{dx}{x_0}G(x,y), \quad G(x,y) = \int_0^\infty F(x,y,t)dt + s_3\frac{\pi}{4}, \\
F(x,y,t) = \mathrm{Im}\left\{e^{f_1(t)}\left[s_2\nu_0^2(1+ib)f_2(t) - s_3 f_3(t)\right]\right\}, \quad b = \frac{4\kappa_1^2}{\nu_0^2}, \quad y = \frac{\varepsilon}{\omega}, \\
f_1(t) = (i-1)t + b(1+i)(f_2(t) - t), \quad f_2(t) = \frac{\sqrt{2}}{\nu_0}\tanh\frac{\nu_0 t}{\sqrt{2}}, \\
f_3(t) = \frac{\sqrt{2}\nu_0}{\sinh(\sqrt{2}\nu_0 t)},
\end{gathered} \tag{16}$$

where

$$s_2 = y^2 + (1-y)^2,\ s_3 = 2y(1-y),\ \nu_0^2 = 4y(1-y)\frac{\omega}{\omega_c(x)},\ \kappa_1 = y(1-y)\kappa(x), \quad (17)$$

ε is the energy of one of the particles of pair, the function $\omega_c(x)$ is defined in Eq.(12) and $\kappa(x)$ is defined in Eq.(4).

In order to single out the influence of the multiple scattering (the LPM effect) on the process under consideration, we should consider both the coherent and incoherent contributions. The probability of coherent pair creation is (see Eq.(12.7) in [3])

$$W^F = \frac{\alpha m^2}{2\sqrt{3}\pi\omega}\int_0^1 \frac{dy}{y(1-y)}\int_0^{x_0}\frac{dx}{x_0}\left[2s_2K_{2/3}(\lambda) + s_3\int_\lambda^\infty K_{1/3}(z)dz\right], \quad \lambda = \frac{2}{3\kappa_1}. \quad (18)$$

The probability of incoherent pair creation is (compare with Eq.(21.31) in [3])

$$W^{inc} = \frac{4Z^2\alpha^3 n_a L}{15m^2}\int_0^1 dy\int_0^\infty \frac{dx}{\eta_1}e^{-x/\eta_1}f(x,y), \quad (19)$$

where L is defined in Eq.(11),

$$f(x,y) = z^4\Upsilon(z) - 3z^2\Upsilon'(z) - z^3 + s_2\left[(z^4+3z)\Upsilon(z) - 5z^2\Upsilon'(z) - z^3\right],$$
$$z = z(x,y) = \kappa_1^{-2/3}. \quad (20)$$

Here

$$\Upsilon(z) = \int_0^\infty \sin\left(zt + \frac{z^3}{3}\right)dt \quad (21)$$

is the Hardy function. For further analysis and numerical calculation it is convenient to use the following representation of the Hardy function and its derivative

$$\Upsilon(z) = \int_0^\infty \sin\left(\frac{\sqrt{3}}{2}z\tau + \frac{\pi}{6}\right)\exp\left(-\frac{z\tau}{2} - \frac{\tau^3}{3}\right)d\tau$$
$$\Upsilon'(z) = \int_0^\infty \cos\left(\frac{\sqrt{3}}{2}z\tau + \frac{\pi}{6}\right)\exp\left(-\frac{z\tau}{2} - \frac{\tau^3}{3}\right)\tau d\tau \quad (22)$$

The probabilities W Eq.(16), W^F Eq.(18), and W^{inc} Eq.(19) at different temperatures T are shown in Fig.1 as a function of photon energy ω. In low energy region ($\omega \leq 1$ GeV) one can neglect the coherent process probability W^F as well as influence of axis field on the incoherent process probability and the LPM effect and the probability of process is $W^{LE} = n_a\sigma_p$ Eq.(8). As one can see in Fig.1 in this energy region the probability W is by 10% at T=293 K and by 20% at T=100 K less than the probability at random orientation W^{ran} which is taken as $W^{ran} = W^{BM}$ (the Bethe-Maximon probability is $W^{BM} = W_0(1-1/42L_0)$=2.17 1/cm in tungsten).

With energy increase the influence of axis field begins and the LPM effect manifests itself according to Eq.(14) (the terms with $\overline{\kappa^2}$ and $(\omega g/\omega_0)^2$ correspondingly). This leads first to not large increase of the probability W^{inc} which attains the maximum at $\omega \sim \omega_m$. The probability W^F in this region is defined by Eq.(6) and its contribution is relatively small. The probability W^F becomes comparable with W^{inc} at $\omega \simeq 1.5\omega_m$. At higher energies W^F dominates, while W^{inc} decreases monotonically.

In Fig.2 the calculated total integral probability W of pair creation by a photon Eq.(16) is compared with data of NA43 CERN experiment [5]. The enhancement is the ratio W/W^{BM}. One can see that the theory quite satisfactory describes data. This statement differs from conclusion made in [5]. One of reasons for this difference is diminishing of incoherent contribution (see Fig.1): for W, $< 111 >$, T=100 K at photon energy $\omega = 55$ GeV one has $W^{inc} = 0.35W^{BM}$, while in [5] it was assumed that $W^{inc} = W^{BM}$.

The contribution of the LPM effect in the total probability W Eq.(16) is defined as

$$W^{LPM} = W - W^F - W^{inc} \tag{23}$$

The relative contribution (negative since the LPM effect suppresses the process) $\Delta = -W^{LPM}/W$ is shown in Fig.3. This contribution has the maximum $\Delta \simeq 5.5\%$ at $\omega \simeq 7$ GeV for T=293 K and $\Delta \simeq 4.3\%$ at $\omega \simeq 12$ GeV for T=100 K or, in general, at $\omega \sim \omega_m$. The left part of the curves is described by the term with $(\omega g/\omega_0)^2$ in Eq.(14). So the rather prevalent assumption that the LPM effect can essentially suppress the pair creation process in oriented crystals is proved wrong due to action of axis field. On the other hand, the LPM effect can be observed in accurate measurements. For observation the LPM effect of mentioned scale in an amorphous tungsten the photons with energy $\omega \simeq 10$ TeV are needed [8].

Acknowledgments

We are grateful to U.Uggerhoj for discussion of experiment and data. The authors are indebted to the Russian Foundation for Basic Research supported in part this research by Grant 03-02-16154.

References

[1] V. N. Baier, V. M. Katkov, and V. M. Strakhovenko, Sov.Phys.JETP **63**, 467 (1986).

[2] V. N. Baier, V. M. Katkov, and V. M. Strakhovenko, phys.stat.solidi(b) **149**, 521 (1988).

[3] V. N. Baier, V. M. Katkov and V. M. Strakhovenko, *Electromagnetic Processes at High Energies in Oriented Single Crystals* (World Scientific Publishing Co, Singapore, 1998).

[4] H. Nitta, M. Khokonov, Y. Nagata, and S. Onuki, Phys.Rev.Lett. **93**, 180407 (2004).

[5] K. Kirsebom *et al*, Nucl. Instrum. Methods Phys.Res. Sect. B **135**, 143 (1998).

[6] A. Baurichter *et al*, Nucl. Instrum. Methods Phys.Res. Sect. B **152**, 472 (1999).

[7] V. N. Baier, and V. M. Katkov, Phys.Lett. A 286, **299**, 2001.

[8] V. N. Baier, and V. M. Katkov, Phys.Rev., **D 62**, 036008 (2000).

Figure captions

Fig.1 Pair creation probability in tungsten, axis $< 111 >$ at different temperatures T. Curves 1 and 3 are the total probability W Eq.(16) for T=293 K and T=100 K, the curves 2 and 4 give the coherent contribution W^F Eq.(18), the curves 5 and 6 give the incoherent contribution W^{inc} Eq.(19) at corresponding temperatures T.

Fig.2

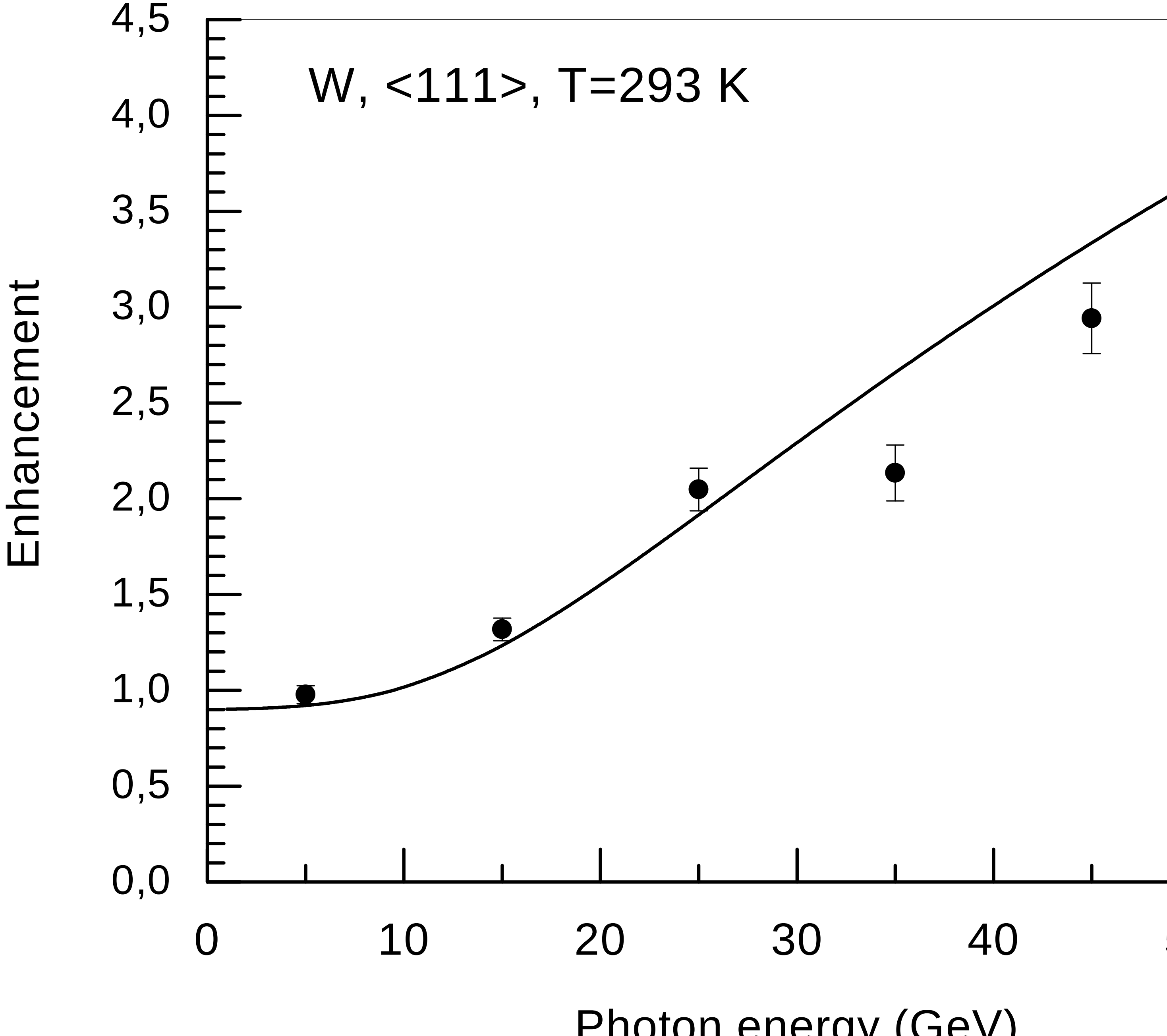
Enhancement of the probability of pair creation in tungsten, axis $< 111 >$. The data are from [5].

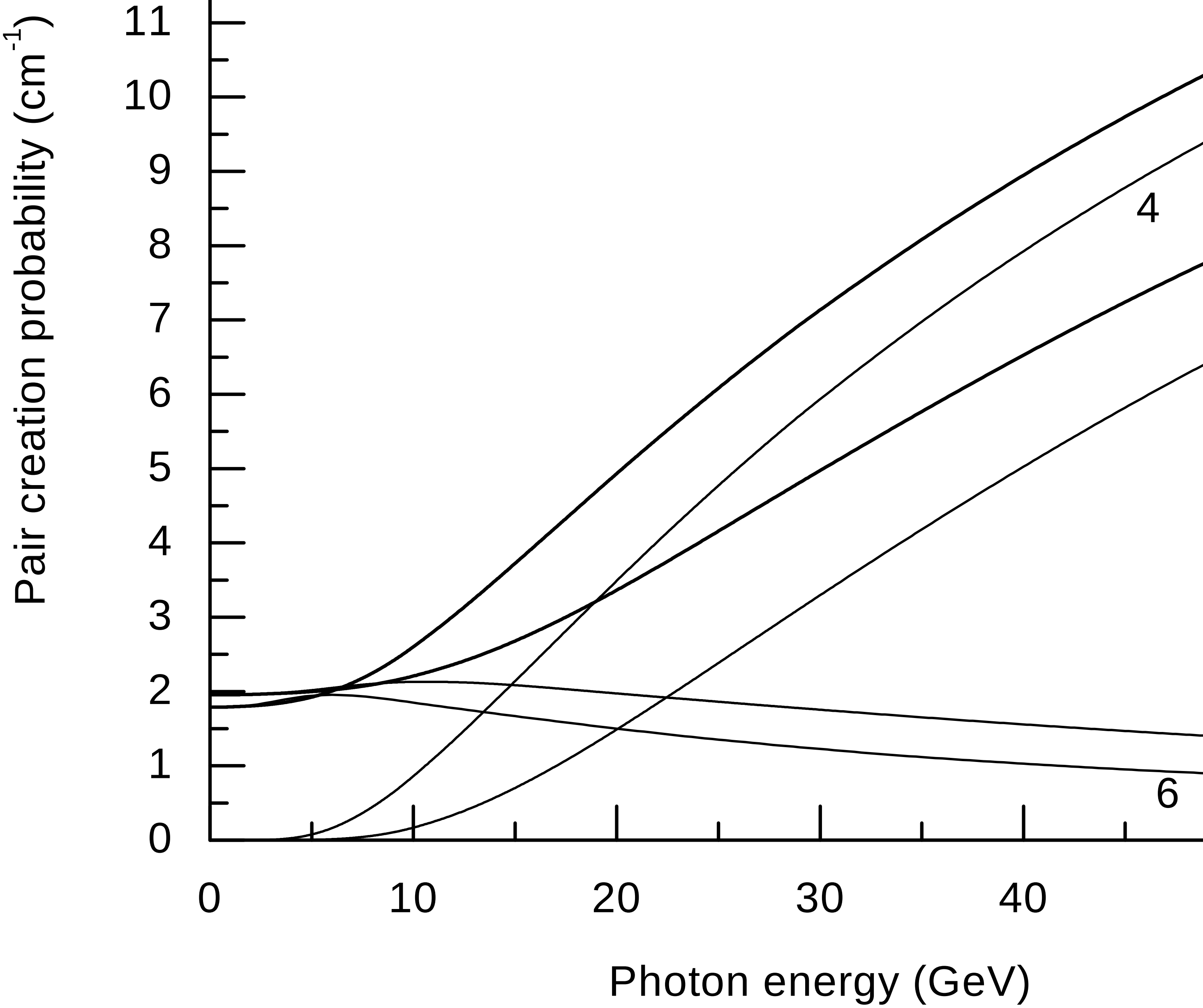
Fig.3

The relative contribution of the LPM effect Δ (per cent) in tungsten, axis $< 111 >$. Curve 1 is for T=293 K and curve 12 is for T=100 K.

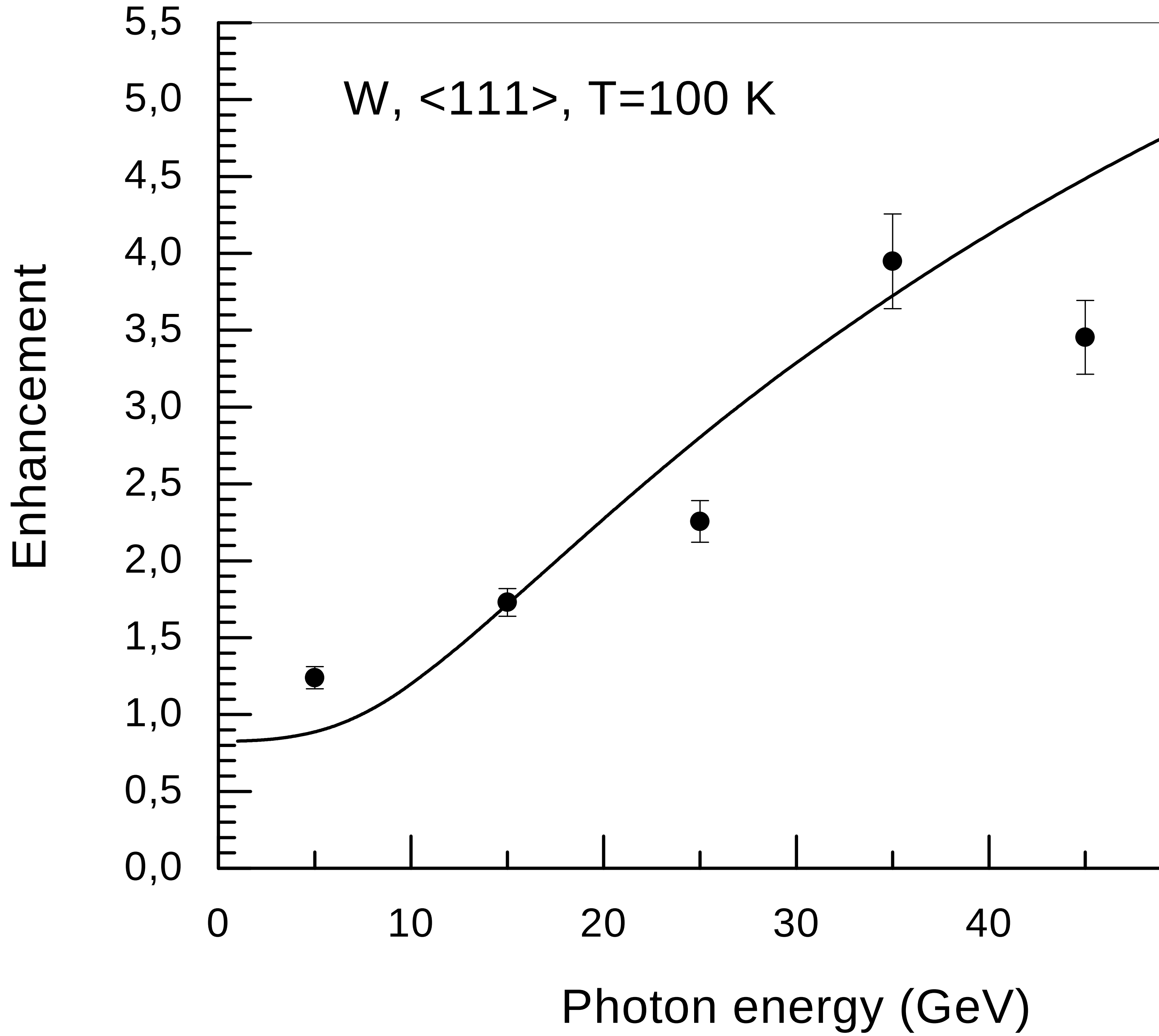
Table 1 Parameters of the tungsten crystal, axis < 111 > for different temperatures T

T(K)	V_0(eV)	$a_s(10^{-8}$cm)	x_0	η_1	ω_0(GeV)	η	ω_s(GeV)	ω_m(GeV)	h
293	413	0.215	39.7	0.108	29.7	0.115	34.8	14.35	0.348
100	355	0.227	35.7	0.0401	12.25	0.0313	43.1	8.10	0.612

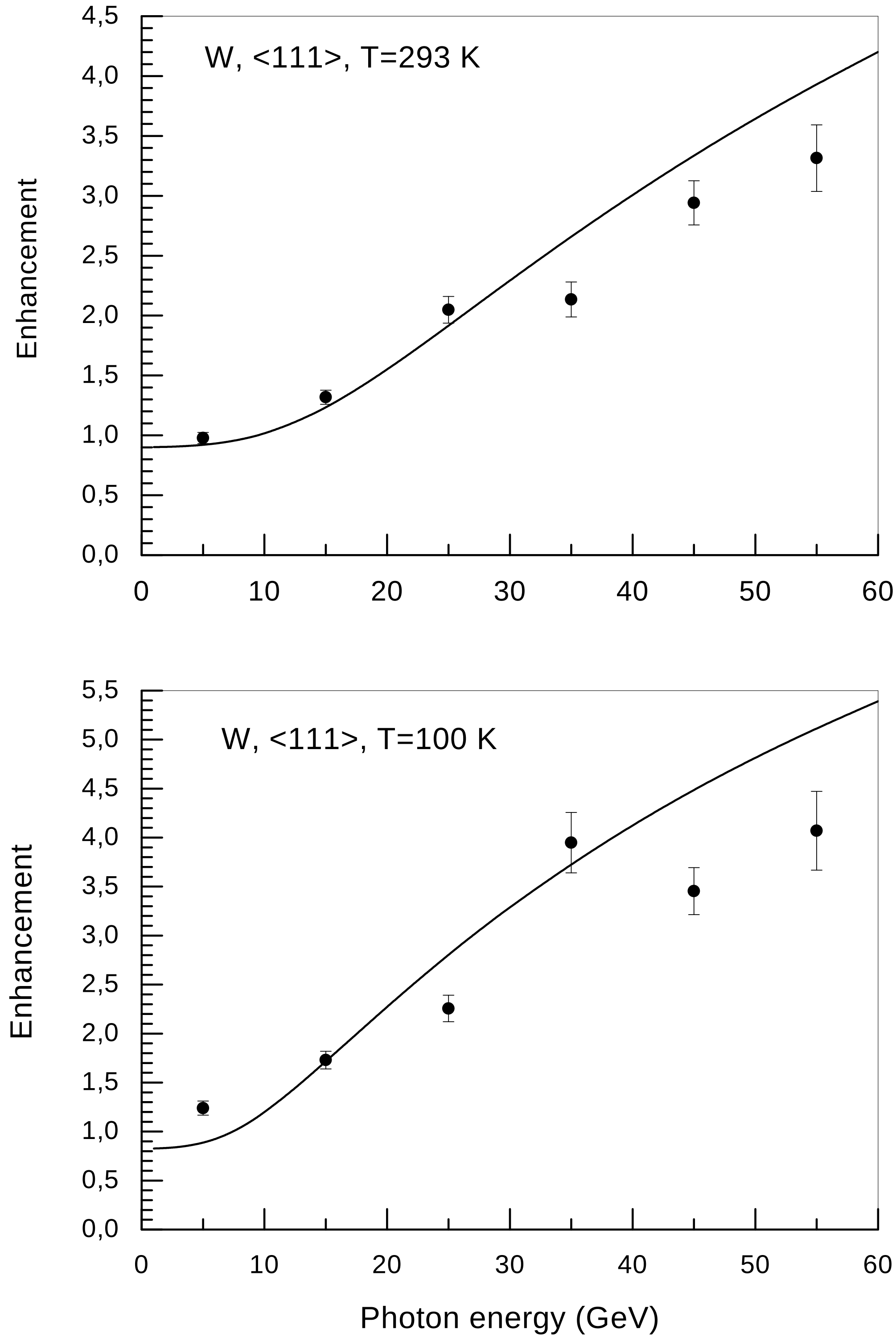

W, <111>, T=293 K
Enhancement
Photon energy (GeV)
4,5
4,0
3,5
3,0
2,5
2,0
1,5
1,0
0,5
0,0
0
10
20
30
40



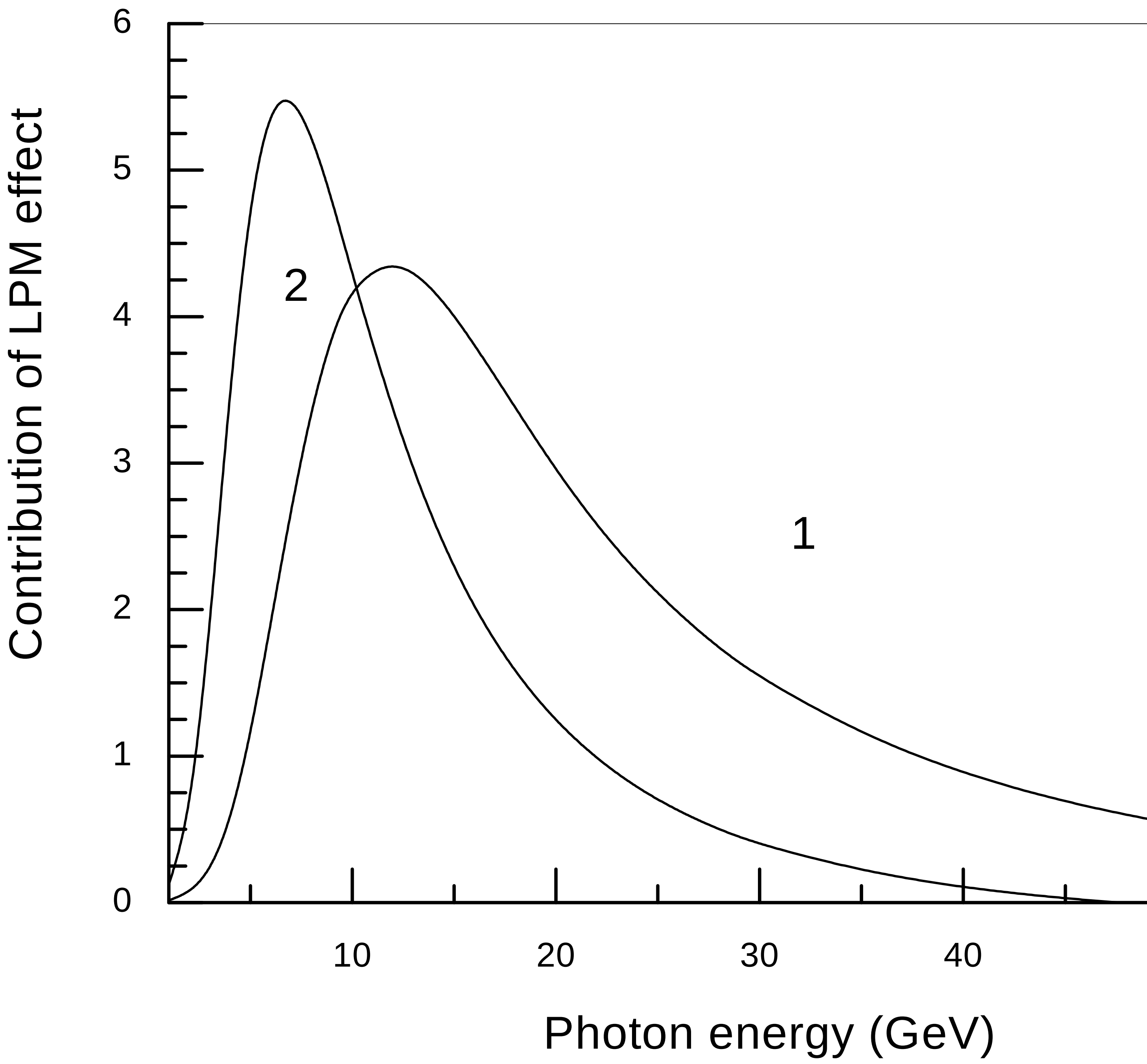
12
11
10
9
8
7
6
5
4
3
2
1
0
Pair creation probability (cm^{-1})
4
6
0
10
20
30
40
Photon energy (GeV)



W, <111>, T=100 K
Enhancement
Photon energy (GeV)
5,5
5,0
4,5
4,0
3,5
3,0
2,5
2,0
1,5
1,0
0,5
0,0
0
10
20
30
40



W, <111>, T=293 K
W, <111>, T=100 K
Enhancement
Enhancement
Photon energy (GeV)
0
10
20
30
40
50
60
0,0
0,5
1,0
1,5
2,0
2,5
3,0
3,5
4,0
4,5
5,0
5,5


Contribution of LPM effect
6
5
4
3
2
1
0
2
1
10
20
30
40
Photon energy (GeV)